

## Law behind second law of thermodynamics — unification with cosmology —

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ABSTRACT: In an abstract setting of a general classical mechanical system as a model for the universe we set up a general formalism for a law behind the second law of thermodynamics, i.e. really for “initial conditions”. We propose a unification with the other laws by requiring similar symmetry and locality properties.

KEYWORDS: Random Systems, Cosmology of Theories beyond the SM, Statistical Methods.

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## 1. Introduction

The second law of thermodynamics [1–3] concerns, contrary to the typical other laws as Newton’s laws in classical mechanics or say the Hamilton equations, the initial state conditions — or better the actual solution among the various possible solutions to the equations of motion — rather than the time development.

It is the purpose of the present article to put forward an attempt to unify the second law of thermodynamics with the other laws, the time development laws [4, 5]. This is done by setting up a more microscopic formulation of this second law of thermodynamics in terms of a postulated “fundamental” probability density  $P(\text{“path”})$  defined for all possible solutions of the equations of motion “path”. A priori we would like to formulate all the laws in the language of microscopic degrees of freedom - these be fields or particles - rather than in terms of the only macroscopically understandable concept of entropy [6].

A good unification of such a probability density  $P(\text{“path”})$  with the time development laws could be to impose the logarithm  $\log P(\text{“path”})$  the analogous type of symmetry and locality properties as the action  $S$ . Even from pure esthetical considerations one could take a completely analogous expression as an assumption or hypothesis for  $\log P(\text{“path”})$  as one writes down for the action  $S$ . If we, for instance, think of a classical field theory with general relativity, the analogous form for the logarithm of the “fundamental” probability

density would be of the form

$$\log P(\text{“path”}) = \int \hat{p}(g_{\mu\nu}(x), \varphi_1(x), \dots, \psi(x), \partial_\rho g_{\mu\nu}(x), \partial_\rho \varphi_1(x), \dots, \partial_\rho \psi(x),) \sqrt{g} d^4x. \tag{1.1}$$

By imposing the same symmetry and locality and perhaps even “renormalizability” restrictions on this  $\log P(\text{“path”})$  as the one on the action  $S$  one will make to an integral expression of the same form, the various coefficients on the various terms be in general quite different for  $\log P$  and for the action  $S$ .

In the beginning of the present article we shall keep the discussion so abstract that we do not even explicitly write down that we are concerned with a (classical) field theory with three space dimensions and one time dimension, but rather just describe the various modes on which we may expand the fields over three space as generalized coordinates  $q_i(t)$  and their canonical conjugate momenta  $p_i(t)$  - we imagine that e.g. spatial Fourier expansion coefficients of the various fields in the field theory describing the universe are taken as the generalized coordinates  $q_i(t)$ . Thus we must imagine that the index  $i$  is a combination and ordered set of symbols some of which denote the type of field and the spin component, while there is in addition a three component wave number enumerating the Fourier coefficients. The  $p_i(t)$  are the canonically conjugate momenta  $p_i(t) = \frac{\partial L(q, \dot{q})}{\partial \dot{q}_i(t)}$ . The space coordinates which are the  $p_i$ 's and the  $q_i$ 's are called phase space and has dimension  $2N$ , where  $N$  is the number of  $i$  values. (Strictly speaking  $N = \infty$  but we will often think as if it were just finite very large.) Thus really the index  $i$  here both describes which field type, which Lorentz or other indices and for which mode the  $q_i(t)$  or  $p_i(t)$  is the expansion coefficient. For simplicity we ignore the fermion fields. In this way we consider the universe in a very abstract manner a general mechanical system, the state of which is described by a point in phase space and the time development. It is given by the Hamilton equations

$$\dot{q}_i(t) = \frac{dq_i(t)}{dt} = \frac{\partial H(q, p)}{\partial p_i}, \tag{1.2}$$

$$\dot{p}_i = \frac{dp_i(t)}{dt} = -\frac{\partial H(q, p)}{\partial q_i}. \tag{1.3}$$

A form of  $\log P$  as given by (1.1) would, by making it into abstract form, become of the form  $\mathcal{P}$

$$\log P(\text{“path”}) = \int \mathcal{P}(q(t), p(t)) dt. \tag{1.4}$$

This general form just could be obtained by imposing a locality assumption in time, which leads to this expression. Then an assumption of time translational invariance is added to make  $\mathcal{P}$  not depend on time explicitly - but only via the variables  $q_i(t)$  and  $p_i(t)$  ( $i = 1, 2, \dots, N$  where  $N$  is the number of degrees of freedom; strictly speaking  $N = \infty$ ).

In the spirit of having in mind essentially random Hamiltonian  $H(q, p)$  as well as random  $\mathcal{P}(q, p)$  we just think of the phase space as a landscape with the two types of heights  $H$  and  $\mathcal{P}$  which may have a priori randomly peaks and dips and passes and so on. In reality they are to obey the symmetry principles that the true laws of nature may

impose on them, but we may at first look for what to expect without putting in too many details, thus avoiding to use these symmetry etc. laws too much.

In the following section 2 we shall put forward how to get an idea for searching for the most likely the highest  $\log(\langle p \rangle e^s)$  — classes of path and see how such considerations lead quite naturally to an effective second law of thermodynamics. At first we only make crude suggestions. But then in section 3 we deliver a very general limitation on how much entropy can go first up and then down and a formal derivation of the second law of thermodynamics follows under the same reasonable approximation. But a further clean up is still needed to apply the second law of thermodynamics even for subsystems. In section 4 we rather shortly put a more concrete cosmological picture on our so far very abstract model. In section 5 we review shortly the various outcomes of the model behind the second law of thermodynamics in addition to the practical outcome of this law itself. In section 7 we conclude and present outlook especially by taking the lack of perfect derivation of the second law of thermodynamics as an extremely interesting suggestion for seeming effects of a foresight.

Such effects could be strange miracle - like events seeking to prevent Higgs particle production, e.g. at LHC.

## 2. Seeking likely class of solutions for realization

### 2.1 Chain of metastable high $P$ regions

We shall seek to get an impression of how to look for the most likely class of solutions if one had at one's disposal the landscape of  $\mathcal{P}$  and  $H$  over phase space.

The most important is in fact to look for regions in phase space in which the system (i.e. the point describing the state of the system  $(q(t), p(t))$ ) can stay around for some time, because one does not get so much contribution from a high  $\log P$  contribution peak in  $\mathcal{P}$ , if the system does not remain in the region so as to get a proper contribution to the integral (1.1). Let us in fact imagine that we investigate the landscape in the enormously high  $(2N)$  dimensional phase space by searching for what we could call metastable regions in which the equation of motion solutions will stay an appreciable time. That would be the regions where the Hamiltonian is near to either a local minimum or a local maximum. Thus A) the partial derivatives of  $H$  are near zero and thus small so that from Hamilton's equation of motion becomes slow and B) there are few or no ways away from it because at a minimum there will not be energy enough to get far away from it and at a maximum there will be too much energy to get away.

Now to get a high likelihood it is crucial that  $\mathcal{P}$  contributes strongly and positively to  $\log P$  ("path") in the neighborhood of such an approximate maximum or minimum region for  $H$ . So we shall really only look for those maxima or minima region with relatively high  $\mathcal{P}$ -contributions to  $\log P$ . Now taking into consideration the huge - really infinite - number of degrees of freedom  $N$  it seems too much to ask for free maxima or minima to occur so copiously that we should expect them with sufficiently high  $\mathcal{P}$  contributions to be relevant. So we will rather think of the metastable regions to be like a minimum say, in  $H$  in by far the majority of the directions in phase space, but not in truly all direction. So

strictly speaking rather a pathlike region with only extremely few coordinates in which  $H$  has maximum, while it is minimal in extremely many coordinates. In whatever way such a metastable regions may come about we may think of crudely ascribing them a “staying time”  $t$  giving the typical time at which the system will spend in that region following the equations of motion. The contribution to  $\log P$  obtainable by having the system “path” such a metastable region called  $A$  is

$$\log P_{\text{contribution from } A} = t_{\text{stay } A} \cdot \mathcal{P}_A \tag{2.1}$$

where  $\mathcal{P}_A$  is the average or typical value of  $\mathcal{P}$  in this region.

Now we are to obtain what are the most likely features of the solution to the equations of motion not simply to ask for that special solution may happen to have the very highest  $\log P$  (“path”). The reason is that the phase space has so enormously many solutions that it could easily happen that there exists some class of solutions with so many of them that even though each of them have significantly lower  $\log P$  than the very highest  $\log P$  solutions the whole class has nevertheless a much higher probability to be realized than this single highest  $\log P$  solution.

When we seek the most likely class of solutions we should thus also estimate the number of solution in the class proposed. We would like to think of this number as an exponential of an entropy  $e^S$ . Here then the entropy for the class of solutions must be related to the thermodynamics entropy concept [7] which we identify as the number of microstates - after discretization with a scale  $U_s = \sqrt{\hbar}$  - in a macro state. The number of solutions in the class cannot be bigger than the entropy at the era, the time, when the entropy is the smallest. There cannot possibly be more solutions than there is phase space place with lowest entropy, in which time they are most compressed in phase space.

Generally we may now look for a class of solutions for which it turns out possible to get the solutions pass through a series of metastable regions  $A$ , i.e., approximate minima or maxima for  $H$  with especially high  $p_A$  and long stay times  $t_{\text{stays } A}$ . For such a class we can achieve a contribution to the logarithm of  $P$

$$\log P |_{\text{contribution}} = \sum_A t_{\text{stay } A} p_A. \tag{2.2}$$

We should further take into account how big a phase space region, or roughly in the discretized way of thinking how many solutions, there of this type, this class. This number is  $e^S$  where  $S$  is the entropy for the lowest entropy one of the regions past provided that we can get organized all the solution passing that metastable state to come through the whole series. It is not a priori guaranteed to be favorable to let it be so. We should in other words look for the entropies of the different regions defined as the logarithms  $S_A$  of the phase space volume in the regions around the approximate and metastable min. or max.  $A$ .

The final formula for the probability of the class becomes

$$P_{\text{class}} = \exp \left( \min_A(S_A) + \sum_A t_{\text{stay } A} p_A \right) \tag{2.3}$$

provided we can get all the  $e^{\min(S_A)}$  through.

We have been crude and worked with the approximation that, although we may work cosmological time unit, these times are still extremely short in any sensible units compared to numbers of the order  $e^S$  where  $S$  is an entropy or compared to the scales for  $P$  variation which are supposed to be similar to those for  $e^S$ .

## 2.2 Macro variables

We should imagine that some combinations of the  $(q_i, p_i)$ 's, i.e. some functions of them (only very few relative to the number of degrees of freedom though) are what we will call “macro variables” such as say the radius  $a(t)$  of the universe or the stand of a piston. Although they may be rather few and although they also have to fulfill the equations of motion they may be relatively so significant for what happens that we should imagine that there would be an effective adjustment of them taking place. That is to say we can imagine that some relatively few macro variables become adjusted in order to make the  $P_{class}$  expressed as an exponent above become as big as possible.

The more contributions in the sum  $\sum_A p_A t_{stayA}$  we can get the higher  $P_{class}$  so there will be a certain “pressure” on the tuning in of the macro variables so as to several metastable regions being passed if possible. Since this passage will have to happen successively in time such a passage of several must mean significant development in time even in a macroscopic sense. Also if we should have coupling constants as in e.g. baby universe theory be considered dynamical variables they should be counted as “macro variables”.

## 2.3 The era simulating big bang with bounce

What can we expect about the entropy at first rather the logarithms of the phase space volumes around the approximate minima (or maxima) of the Hamiltonian? It may depend on many details but a priori these volumes could be quite different even in logarithm - which is what we call the entropies -.

Thus we can a priori expect some entropy variation with time, at least there seems a place for it, a first ingredient for possibly obtaining the second law of thermodynamics.

The metastable region  $B$  with the smallest entropy  $S_B$  is the lowest phase space  $e^{S_B}$  in a series of such metastable regions  $A$  passed by the dominating class of solutions. It is the one with a relatively low  $t_{stayB}$  and a very high  $p_B$  because we would statistically expect to get  $\mathcal{P}_B$  which is an average over the region  $B$  huger the smaller this region i.e. the smaller  $S_B$ .

Here we should think a balance among the contributions so that under the variation of various macro variables,  $\xi$ , we should have

$$0 = \frac{\partial \log P_{class}}{\partial \xi} = \frac{\partial S_B}{\partial \xi} + \sum_A \frac{\partial}{\partial \xi} (t_{stayA} p_A) \tag{2.4}$$

This should be true for all the “initial” macro variables. This fact suggests some compensations that all the  $t_{stayA} p_A$  have got about the some orders of magnitude. So if  $\mathcal{P}_c$  large then presumably  $t_{stayC}$  small relatively.

But really it does not matter, one of the contribution will have the smallest volume for its metastable phase space region and we call it  $B$ . The era of  $B$  we then take as a middle point in time  $t_o$  which by additive shift of the time axis we can put to zero  $t_0 = 0$ .

We shall now suggest the interpretation that in usual cosmology we ignore the times on the one side of this time  $t_0 = t_B$  for the staying around the lowest entropy region  $B$  and shift the sign convention so that we live after this era of  $B$ . The era before  $B$  we ignore as “pre-big bang era” and not taking it seriously. Now we are in somewhat better setting to obtain the second law of thermodynamics because we have made the lowest entropy metastable state in the series. Because it becomes a first era taken seriously. With the lowest  $S = S_B$  to begin with we have at least better chance for increasing away.

### 3. Derivation of second law of thermodynamics

To really get a formal argument that the entropy has to increase all the time we shall make use of a little lemma restricting strongly the possibility for a too big maximum in entropy as a function of time. Such a theorem is only of interest when we have a discussion without the second law of thermodynamics, because the latter not only restricts maxima but also totally forbids them in as far as  $S(t)$  becomes monotonically increasing and thus cannot have maxima at all.

We have indeed a theorem alone from the equations of motion assumed to be not fine tuned saying: If a system - with random Hamiltonian - has in time the successive entropies considered phase space volumes relative to a cut off unit  $u_s^{2N} = h^N$  say,  $S_1, S_2$  and  $S_3$  under a “reasonable time interval”, then we have

$$S_2 \leq S_1 + S_3. \tag{3.1}$$

The requirement of the “reasonable time scale” means that the time interval involved is so small that the number of cells of size  $u_s^{2N}$  past during it is exceedingly small compared to exponentiated entropy numbers  $e_1^s, e_2^s, e_3^s$ .

This theorem is proven by considering that the number of cells in the macro state with the middle time entropy  $S_2$  which will reach the  $S_3$  entropy macro state can at most be  $e^{S_3}$ , apart from an unimportant reasonable factor which makes up a fraction  $e^{S_3-S_2}$  of all the states there. Statistically the fraction of the  $e^{S_1}$  cells into which the cells in the first macro state - the one with entropy  $S_1$  - can come, which can go on to the third macro state is thus  $e^{S_3-S_2}$ . This means that  $e_1^S \cdot e^{S_3-S_2}$  cells come through all three macro states, but that is more than one for  $S_1 + S_3 - S_2 \geq 0$ , which is just the condition we were to prove. This theorem seems essentially to disagree with the two entropies idea propose by one of us (H.B.N.) and L.E.Rugh [8] following Hartle and Hawking [9].

Combining this theorem with the hypothesis that the minimal entropy  $S_B$  metastable region passed in the development is very small compared to the later ones (or better the ones with higher  $|t - t_0|$  i.e. further out) at least some of them, we get the second law of thermodynamics. Indeed denoting the entropies at times  $t$  and  $t'$  by  $S(t)$  and  $S(t')$  and assuming  $t'$  further out than  $t$  i.e.  $|t' - t_0| > |t - t_0|$ , and on the same side of  $t_0$  we can apply our theorem on the three successive times  $t_0 < t < t'$  or if  $t$  and  $t'$  are on the other

side  $t' < t < t_0$ . The theorem then gives the condition

$$S(t) \leq S(t_0) + S(t') = S_B + S(t'). \tag{3.2}$$

With the extra hypothesis that  $S_B$  is exceedingly small we get

$$S(t) \leq S(t'), \tag{3.3}$$

which is the second law of thermodynamics and we are through deriving it.

How likely is it, however, that  $S_B$  is indeed so small?

It is not so unrealistic that the metastable region  $B$  with the smallest entropy is really not metastable at all, but rather only one cut off cell of volume  $u_2^{2N} = h^N$  with an extremely high  $\mathcal{P}$  value so that even from the passage of about one cell there comes a significant contribution to  $\int \mathcal{P} dt$  from it. If  $\mathcal{P}$  has enough variation at small distances in phase space such a happening is not unlikely.

#### 4. How to see our model more concretely in cosmology

How does this abstract picture of a universe passing through a series of or at least some metastable macro states with especially high  $p$ -contribution match with the phenomenological picture of the universe development? One thing which is usually assumed in realistic field theoretical model — on grounds which are mainly phenomenological though — is that there is a bottom in the Hamiltonian. That feature we can now consider among the predictions of the present model behind the second law of thermodynamics.

Perhaps one does not always appreciate how much could have gone wrong with this bottom property because one would tend to forget that also a wrong sign on the coefficients of the kinetic terms in the Lagrangian density for a boson field  $\frac{1}{2}(\partial_u \varphi)^2 + m^2 \varphi^2$  easily could have spoiled the bottom. The fact that different gauge fields in nature indeed have the same sign on the kinetic terms is non-trivial and a remarkable property for a non-simple Lie algebra or rather Lie group theory. Concerning simple Lie groups a lot of them should be discarded on the grounds of such a positivity requirement for the Hamiltonian, but Nature has chosen just the compact Lie groups suitable for having a bottom!

It should be remarked however that with respect to gravity we cannot say that we have a bottom in the energy density or Hamiltonian density. Strictly speaking it is a complicated story even to define what we should identify as energy [10], [11] to get a sensible one. A slightly different point of view on energy in general relativity is to consider some of Einstein equations or, if we think quantum mechanically (what we do not take into account of in the present article), the Wheeler-de Witt equation that as a constraint total energy density gravitational plus matter energy density is zero. This point of view is analogous to absorbing the  $div D$  in the Maxwell equation  $div \vec{D} = \rho$  into the charge density and declaring that this Maxwell equation is a constraint that tells that the “total charge”  $\rho - div \vec{D}$  is zero, considering  $-div \vec{D}$  a sort of “electromagnetic extra charge”



analogous to the “gravitation energy density.” With latter point of view the gravitational energy is just equal with opposite sign to the matter energy and we have no bottom in it.

Now, however, since gravity is under present circumstances a weak interaction the bottomlessness due to gravity may not severely threaten the metastability. Realistically we might think of some global or perhaps for limited region crunch of the universe. In such a case so strong gravity fields may appear and the bottom is (effectively) lost. In the light of the facts of that gravity spoils a perfect bottom but that there is a good approximation a bottom in the matter part of Hamiltonian it seems that our present universe fits wonderfully to a metastable region. The stability may also correspond to an extremely long stay time  $t_{stay}$ . At least we have stayed in a similar state for of the order of 13 milliard years.

The discussion in the last section suggested that the middle era around the region  $B$  with the smallest entropy  $S_B$  — taken in our interpretation as the earliest era — be rather unstable than truly metastable. It should be a state with an especially high  $p$  selected, without too much caring for the metastability. This could mean in the General Relativity analogous ansatz form that a combination of scalar field values should be used with the “potential part” of  $\hat{\mathcal{P}}$ , especially probability favorable. That is to say that the scalar fields should be near a maximum in the latter potential part of  $\hat{\mathcal{P}}$ . This means that some value combination should be used all over the universe if, as the ansatz suggests, we assume the  $\hat{\mathcal{P}}$  translational invariant in both space and time.

This leads to an inflation period [12] of the translational invariant type rather than of the chaotic type. In this way we can claim, within only natural assumptions, to predict homogeneous inflation. Although it will clearly be highly favorable to probability to keep the scalar fields extremely slowly rolling down the potential, it is not so obvious if it can indeed be organized by setting variables in the “initial state” - because in fact the rolling rate is determined by the equations of motion and the mechanical potential  $V_{eff}(\varphi_1, \dots)$  in the Lagrangian  $L$ . Unless the manipulation so as to optimize  $\langle P \rangle e^S$  as discussed above can be extended to become also a manipulation of the coupling constants determining the inflation effective potential the optimizing  $\langle P \rangle e^S$  will, however, not be able to adjust the potential form as is needed to ensure slow rolling. But whatever this optimization has to organize will be organized to obtain the slow rolling. So if there is any chance our model goes in the direction of doing what it can solve the slow rolling problem.

It should be stressed as an interesting and presumably unexpected feature that our model does not favor a true big bang singularity, but rather that as just discussed above an as slowly rolling as it can organize it is singularity free inflation even at the midpoint  $t = t_0$  of time, in the middle of era  $B$  in the above notation. Actually our picture is a de Sitter model with positive spatial curvature - to keep the universe finite and give a finite positive turning point radius - which before  $t_0$  contracts, reach a finite value at  $t_0$  and then expands first slowly and faster and faster dominated by the effective cosmological constant from the effective potential around the scalar fields close to the very  $\hat{\mathcal{P}}$ -favored value-combination. Our scenario is thus smoother than the usual one with true big bang

singularities. The very highest Planck temperatures may never have been realized in our scheme!

## 5. Some results and predictions

We have already written about several outcomes of the model in the foregoing sections in addition to the derivation of the effective raising of the entropy on our side of the middle, our region of the  $t$ -axis. We suggested that entropy raises as one goes along the time axis away from “the middle point”  $t_0$  (called 0) roughly characterized as the era  $B$  with the smallest entropy  $S_B$ . In fact we found in addition the fact that we should have an approximate bottom in the Hamiltonian density so as to ensure metastability to make the most possible use of high  $\mathcal{P}$  around the state of the universe as was actually found. Further we managed as suggestive outcome, from longer time scales, to see effective big bang although we do not have a genuine singularity but rather a smooth de Sitter scenario; first contracting - but with inverted second law of thermodynamics - and then bouncing and expanding, inflating.

As we have already touched upon a bit it would be very tempting to talk in this language about the  $P$  optimization as if it were a person with will and the  $P$ -governing of the solution if it could also influence the coupling constants in the Lagrangian. That could easily turn out to be helpful to get a higher likelihood chance of tracks. In some theories like baby universe theory, [13]-[16] such influence can come as a consequence of what is effectively outside in our phenomenological universe theories - the baby universes - while in others you can imagine some effective dependence of the effective couplings on flat directions or the like that. Also in formulating the fundamental theory one could just choose to include the couplings, mass parameters and kinetic term coefficients  $Z_i$  under the dynamical variables so that they are allowed to have the possibility of having many - all real values - as possible values. In this way the  $P$ -probability machinery could come to determine also the values of the coupling constants etc...

Whatever the excuse or the mechanism behind we already alluded to the fact that allows the parameters in the inflation potential to be “dynamical” and thus to be allowed to participate in maximizing  $\langle P \rangle e^S$ , it would make them tune in favor of a more slow rolling during inflation. This might solve the problem of the slow rolling problem requiring such an unnatural and strangely tuned potential. It was indeed tuned according to our model. Perhaps there is even some  $P$ -benefit by having non-zero but some small density of matter in the universe at a certain time!

May be the best is to suggest that by 50% chance at least there is a making bigger universe radius  $a(t)$  more unlikely than smaller ones. Then a balance may appear. A most interesting result that can suggestively come out once the coupling constants etc. are taken to be “dynamical” (meaning that take different values) is the solution of the famous cosmological constant problem: Why is the dressed or effective cosmological constant so extremely small in Planck units? Assuming that there is some term in  $\mathcal{P}$  making it favorable to keep  $a(t)$ , the universe radius not too big we may argue for a slow expansion of the universe radius. Indeed it is very important for preventing from crunching or collapse

via black holes that the matter and radiation density be sufficiently low so that big local crunches do not take place. So to keep it proper metastable there is a need for big size of the universe so as to get the density down to prevent from crunches or collapses via black holes. However these crunch dangers fall to exceedingly low probabilities already with today's densities. So further reduction of the density is not needed for exceedingly long time already. Thus with the assumed term seeking to keep the universe radius from going to infinity is there - at least 50% chance - then the radius will settle to grow slower and slower. Since also density must be low in the assumptions of radius itself already large one can simply, from the Friedmann's equation, see that a small cosmological constant  $\Lambda$  is needed. If thus  $\Lambda$  is adjustable it will be put small! This is a solution to the cosmological constant problem in our theory presented in this article. It should, however, be admitted that the best attempts to solve the cosmological constant problem in literatures [17], [18] already went this way of making it effectively dynamical one way or the other and then the most important part of solving the cosmological constant problem is over.

We must, however, stress that a complete solution of this famous problem [19] must also contain what then determines the value of such a “dynamical” cosmological constant just to be small. In principle if it has become “dynamical” dependent on dynamical variables, on initial conditions so to speak, one needs a “theory of initial conditions” to settle the value to be chosen. Here it should be mentioned that since the cosmological constant  $\Lambda$  - or any other coupling constant or mass parameter (=bare mass) is constant as function of space and time, there is no special meaning in assigning it them to any special moment of time, like there is for other dynamical variables. It is therefore a priori not clear how one should specify coupling constants as “initial conditions” unless one has as we proposed in the present article a more detailed model for what “initial conditions” shall be.

In this sense we can claim that this sort of behind the second law of thermodynamics law is needed whenever a solution to the cosmological constant problem is sought by making the cosmological constant  $\Lambda$  a “dynamical” variable, a part of initial condition. On the other hand one can hardly hope to solve the cosmological constant problem without making the cosmological constant in some way or the other depend on the happenings in universe. So it almost has to be “dynamical” and our type of model is thus needed!

Let us finally mention that we hope in a soon to come article [20] to derive from the present model the more detailed result of what we have called the multiple point principle [21] and one of us (H.B.N.) and his collaborators worked a lot on from a more phenomenological point of view. This multiple point principle which we seem to be able to derive from the model says that there are many minima in the scalar field effective potential that are all going to be fine tuned into having almost the same cosmological constant or potential height, in the present model essentially zero, from Planck scale point of view.

## 6. How the dependence on the future got diminished

The arguments given above for the second law of thermodynamics were not very convincing, because the solution probability density *does* depend on what the track does at all times, also on times later than the time at which one performs some test of the second law of

thermodynamics. Therefore one might imagine that some compromise could come up and some adjustment take place to reach from the moment view point of future high  $\log P$ -contribution. That is what would look like pre-arrangements. The question really is why such pre-arrangements are not much more common experimentally.

There is indeed an argument that such selection of the solution influenced by the future will be suppressed. In fact we shall see by arguing that under the conditions for which we already argued to occur in the asymptotics the probability density will be almost constant as a function of the behavior of the solution “*path*” in this future.

Then we argued for the asymptotic situation to have the properties - already fulfilled in the present time of the universe development -:

1. Essentially only massless particles
2. Lorentz invariance
3. Low density  $\Rightarrow$  little interactions

In order to estimate that  $P$  is indeed practically constant as a function of the behavior of the solution in such a part of the time, instead of treating the universe by field theory as a classical approximation, we shall rather think of particles (quanta). Of course at the end we have the belief that the theory should be treated quantum mechanically, but we can approximate a quantum field theory by a classical theory in different ways; we can either treat it as a theory of classical fields or as a theory of classical particles. These two are not the same approximation and in general only good approximations under different conditions.

The crux of the matter is that if we should write an analogue to the field theory expression for the particle classical approximation, then the free particle contribution to the  $\log P$ (“*path*”) would have to be a sum over all particles of integrals along the time track of these particles in space time. The quantity to be integrated along the time tracks can only depend on the internal quantum numbers and polarization of the particle in question, while rotational invariance grounds it cannot really depend on polarization. The expression would be of the form

$$\log P(\text{“path”}) = \sum_i k_i \tau_i + \text{“interaction contributions”} \quad (6.1)$$

where the sum runs over the individual particles  $i$ , and  $\tau_i$  is the eigentime for the particle from it were created till it gets annihilated, and  $k_i$  a number characteristic of its species and internal quantum setting.

The point is that if there remain mainly massless particles then their eigentimes are always *zero*. So the contributions from massless particles are indeed negligible except for the contributions when they meet and can interact. But if interaction is also negligible in the future times then even such interaction caused contributions will be small. Under these condition,  $P$  is practically constant as a function of the behavior of the solution in era like the asymptotic one!

Provided the future is of this nature then we deduce the following statement:

No adjustment to future, i.e. no hand-of-God-effects.

What we see with 2<sup>nd</sup> law of thermodynamics, i.e. no regular future, is thus just what comes out when the future due to dominance of massless particles is not influential on the choice of the solution “*path*” thus leaving it for the inflation time physics to decide.

The formal derivation, if you have physical conditions are o.k., may sound like this:

Why is  $S$  in the short time middle state appreciably lower than outside? May be we can argue in the following way: There will likely be enormous peaks in  $p(t)$  (or  $P$ ) so that even a short stay in time around such peak could be of significance to bring over all likelihood up.

But most likely a high peak is a narrow peak - as we would expect for a reasonable ansatz for a “random” function  $p$ - the more you average, then less outrageous average. Thus to get use of a high peak presumably a very low entropy only can be allowed. This means  $S$  must at least increase away on average from this special time  $t_0$ . A truly useful 2<sup>nd</sup> law of thermodynamics has to work for sub-systems separately. Recall our theorem against entropy in middle more than twice average of side times.

Could we extend the theorem also to work more locally or rather for subsystems? Of course yes.

## 7. Conclusion and outlook

We have, in the present article, put up a very general formalism for how one way formulate a law behind the second law of thermodynamics, or we could say more generally a law for which actual solution to the equations of motion are the more likely. In guessing such a law it is natural to seek to impose on the probability  $P$  (“path”) the symmetries of the other - i.e. time development - laws, even time reversal invariance and locality in time (and space), but that would a priori look like causing problems in obtaining the second law of thermodynamics.

We therefore consider it a quite remarkable result that we, in spite of this a priori unpromising outlook, in fact argue for the second law of thermodynamics to come out, even without to specific assumptions about the probability density functional  $P$  in our “model”. Here “model”, really the formalism is so general that there is only very little content, until one helps by adding for a speculations about properties of the functional  $P$ ! Nevertheless we got second law of thermodynamics with only very mild such further speculative assumptions, essentially we got it for “random” functional  $P$  with some locality in time and time translational invariance.

The most interesting is that we only obtain it approximately and in practice, but not totally exactly. Especially since in our model the probability density functional also depends on the behavior of the solution to the equations of motion in the future, the model is a priori likely to predict effects of seemingly foresight or the Hand-of-God. Indeed the prediction of an effective bottom in the Hamiltonian in practice and even of a small cosmological constant, and the multiple point principle [21] (see in section 5 at the end of the present article) may be considered such foresight arrangements. In this sense we

have already in this article discussed how some such foresight effects are indeed successful predictions for our model. Since also an approximate - and good enough - big bang phenomenology with homogeneous inflation comes out as a prediction we can consider the predictions very successful. There is though in our picture no big bang singularity [22].

One could consider any thought of foresight effect meaning that something in the present is adjusted to some special or simple coincidence in future as a violation of the second law of thermodynamics. If some features today lead to a special state tomorrow it means that ordering some times in some cases increase as time goes on which is just against the second law which says that order gets less and less with time. Our model is a priori filled with such foresight effects and our formal proof of second law of thermodynamics only means that we crude by get rid of them. So we do not predict that such miraculous effects are totally excluded in our model. A support our model over a more pure and exact second law of thermodynamics would precisely be if such prearranged Hand-of-God effect shows up convincingly. We claimed that large universe radius low density bottom in Hamiltonian are already in our model of the Hand-of-God type.

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